Speaker: Géza Tóth
Title: The Erdős-Szekeres Theorem, and a generalization for lines

Abstract: According to the Erdős-Szekeres Theorem (1935), for every $n$ there is a (smallest) $f(n)$ such that any set of $f(n)$ points in the plane (in general position) contains $n$ in convex position. Erdős and Szekeres proved that

$$2^{n-2} + 1 \leq f(n) \leq \left( \frac{2n - 4}{n - 2} \right) + 1 \approx 4^n / \sqrt{n}.$$  

(Then Szekeres married Esther Klein, who posed the problem.) The lower bound has not been improved and it is conjectured to be the truth. Recently the upper has been improved a little bit, to $\binom{2n-5}{n-2} + 1$ in several steps. We review these improvements, some related problems, and then we investigate the “dual” version of the problem.

We say that $n$ lines are in convex position if they bound a convex $n$-gon. Let $f_l(n)$ be the smallest number with the property that this many lines in general position always contain $n$ in convex position. We show a lower and upper bound for $f_l(n)$ of roughly $4^n / n$ and $4^n / \sqrt{n}$, respectively. So we have a much better lower bound for lines then the one for points.

We also try to “dualize” some of the generalizations, different versions of the original Erdős-Szekeres Theorem.

Joint work with Imre Bárány and Edgardo Roldán-Pensado.